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## FORECASTING USING FTS

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### Abstract

In Fuzzy time series forecasting, forecasting are mostly affected by length of the intervals. In this paper, the rule for determining the length of interval is proposed. Based on the new intervals, weightage factor and fuzzy logical relationship, more accuracy results are got than the existing methods in the historical enrollments of the University of Alabama and the comparison results are given.

**Keywords:** Fuzzy Time Series, Fuzzy Sets, Fuzzy logical relationships, Fuzzy forecasting, Weightage factor.

**AMS Mathematical Subject Classifications:** 03E72, 62M10.

### I. Introduction.

Song and Chissom (1993,1994) proposed time variant and time invariant methods whose observation are linguistic values. They asserted that all traditional forecasting methods not suitable when the historical enrollment data are composed of linguistic values, Forecasting using fuzzy times series has been widely used in many activities. The definitions and properties of fuzzy time series forecasting are found in Zadeh(1975), Wu(1986) and Song and Chissom(1993,1994). Sulliwán and Woodal(1994) described a Markov model using linguistic values directly but with membership function of the fuzzy approach replaced by analogous probability function. In the place of complicated maximum – minimum composition operators Chen (1996) used a simple arithmetic operation for time series forecasting. Hwang , Chen and Lee(1998) presented a method of forecasting enrollments using fuzzy time series based on the concept that the variation of enrollment of this year is related to the trend of the enrollments of the past years. Huarng(2001) indicated that the length of intervals will affect the forecasting accuracy rate and a proper choice of length of the intervals can enhance the forecasting result. He presented the distribution length approach and the average based length approach to deal with forecasting probabilities based on the intervals with different lengths. Chen(2002) presented a method of forecasting based on high-order fuzzy time series. Chen and Hsu(2004) proposed a first order time variant method for enrollment forecasting using fuzzy time series. Also M. Sah and Y.D Konstantin(2005) presented a first order fuzzy time series method of forecasting. Chen et.al(2009) presented clustering techniques for clustering historical enrollments in to intervals of different lengths. Lee.et.al(2009) proposed the modified weighted method for enrollment forecasting. Wang and Chen(2009) presented a forecasting method based on clustering techniques and two factors higher order fuzzy time series.

In order to get a higher forecasting accuracy rate, in this paper, a new attractive simple method is presented to forecast the enrollments of the University of Alabama based in interval length, weightage factor and fuzzy logical relationships

### Fuzzy Time Series

Some concepts from Song and Chissom (1993,1994) are reviewed. The difference between the fuzzy time series and conventional time series that the value of the former are fuzzy sets while the values of the later are real numbers.

Define the Universe of Discourse  $U$ , find the minimum enrollment  $D_{\min}$  and the maximum enrollment  $D_{\max}$  from the known historical data. Based on the  $D_{\min}$  and  $D_{\max}$ , define  $U$  as  $[D_{\min} - C_1, D_{\max} + C_2]$  where  $C_1$  and  $C_2$  are the two positive numbers. Partition the universe of discourse in to intervals  $u_1, u_2, \dots, u_n$ .

A fuzzy set of  $U$  is defined by  $A = \frac{\mu_A(u_1)}{u_1} + \frac{\mu_A(u_2)}{u_2} + \dots + \frac{\mu_A(u_n)}{u_n}$  where  $\mu_A$  is the membership function of  $A$ ,  $\mu_A: U \rightarrow [0,1]$  and  $\mu_A(u_i)$  indicates the grade of membership of  $u_i$  in  $A$ , where  $\mu_A(u_i) \in [0,1]$  and  $1 \leq i \leq n$ .

**Definition:** Let  $Y(t)$  ( $t = \dots, 0, 1, 2, \dots$ ), a subset of  $\mathbf{R}$ , be the universe of discourse on which fuzzy sets  $f_i(t)$  ( $i = 1, 2, 3, \dots$ ) are defined and let  $F(t)$  be the collection of  $f_i(t)$  ( $i = 1, 2, 3, \dots$ ). Then  $F(t)$  is called a fuzzy time series on  $Y(t)$  ( $t = \dots, 0, 1, 2, \dots$ )

We see from the definition that  $F(t)$  can be regarded as a linguistic variable and  $f_i(t)$  ( $i = 1, 2, 3, \dots$ ) can be viewed as possible linguistic values of  $F(t)$ , where  $f_i(t)$  ( $i = 1, 2, 3, \dots$ ) are represented by fuzzy sets. If  $F(t)$  is caused by  $F(t-1)$  only, then this relationship is represented by  $F(t-1) \rightarrow F(t)$ . If  $F(t-1) = A_i$  and  $F(t) = A_j$  where  $A_i$  and  $A_j$  are fuzzy sets, then the fuzzy logical relationship between  $F(t-1)$  and  $F(t)$  can be represented by  $A_i \rightarrow A_j$  where  $A_i$  and  $A_j$  are called the current state and the next state of the fuzzy logical relationship respectively.

### Length of Interval

For enrollment forecasting, Song and Chissom (1993, 1994) choose 1000 as the length of intervals without specifying any reason. Since then 1000 has been used as the length of intervals in further studies. In fact, different length of intervals may lead to different forecasting results. Kunhuang Huarng (2001) presented a method in which the length of the intervals is 400. Chen et al (2009) presented a method for forecasting the enrollments of the University of Alabama based on automatic clustering techniques and fuzzy logical relationships. The automatic clustering techniques generate 21 intervals with different length of intervals. Vamitha et al (2015) presented the technique of dividing the intervals in to 18 unequal length of intervals. However the Mean square error (M.S.E) is very less compare to other methods when 21 intervals are divided in to 105 intervals with different length. This is the motivation to propose a simple method to generate intervals with even length.

**Step 1:** Sorting the actual enrollment in an ascending sequence.

**Step 2:** Obtain the difference between the adjacent values.

**Step 3:** Compute the length of interval as

$$\text{Length of interval} = \frac{\sum_{i=1}^{n-1} (x_{i+1} - x_i)}{n-1}$$

Where  $n$  is the number of difference.

### Proposed Method

In this section, a new method for forecasting enrollments based on the proposed length of interval, weightage factor and fuzzy logical relationships is presented.

**Step 1:** Apply the proposed length of interval to obtain the intervals and calculate the midpoints of each interval. Add the calculated length with the first value of the sorted sequence gives the first interval. With the end point of the first interval add the calculated length gives the second interval. Proceed up to the last value of the sorted sequence.

**Step 2:**

Assume that there are n intervals  $u_2, \dots, u_n$ . Then using trapezoidal membership rule define each fuzzy set  $A_i, 1 \leq i \leq n$  as follows:

$$A_1 = \begin{matrix} & \dots & \dots & \dots & & & \\ & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \downarrow & \downarrow & \downarrow & \uparrow & \uparrow & \\ & \dots & \dots & \dots & & & \\ & \dots & \dots & \dots & & & \end{matrix}$$

$$A_2 = \begin{matrix} & \dots & \dots & \dots & \dots & & \\ & \uparrow & \uparrow & \uparrow & \uparrow & \downarrow & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \uparrow & \\ & \dots & \dots & \dots & \dots & & \\ & \dots & \dots & \dots & & & \end{matrix}$$

$$A_3 = \begin{matrix} & \dots & \dots & \dots & \dots & \dots & \\ & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ & \dots & \dots & \dots & \dots & & \\ & \dots & \dots & \dots & & & \end{matrix}$$

⋮

⋮

⋮

$$A_n = \begin{matrix} & \dots & \dots & \dots & \dots & \dots & \\ & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ & \dots & \dots & \dots & \dots & & \\ & \dots & \dots & \dots & & & \end{matrix}$$

**Step3:**

Fuzzify each data in the historical enrollments in to a fuzzy set. If the data belongs to  $u_i$ , where  $1 \leq i \leq n$ , then the data is fuzzified in to  $A_i$ .

**Step 4:** Construct the fuzzy logical relationship (FLR) based on the fuzzified historical enrollments obtained in step3. If the fuzzified enrollments of year's  $t$  and  $t+1$  are  $A_i$  and  $A_j$  respectively, then construct the fuzzy logical relationships  $A_i \rightarrow A_j$  where  $A_i$  and  $A_j$  are the current state and the next state of fuzzy logical relationships respectively. Divide the fuzzy logical relationships in to fuzzy logical relationship group(FLG), where the fuzzy logical relationships having the same current state are put into the same fuzzy logical relationship group.

**Step5:**

We consider the weighted factor [Yu(2005)] and the difference between the actual data and the mid values of the intervals. When  $A_i \rightarrow A_j^{(1)}, A_k^{(1)}, A_l^{(1)}, A_m^{(1)}$  be a fuzzy FLG, assign numbers as follows:

$$w_j = \frac{j}{j+k+l+m}, w_k = \frac{k}{j+k+l+m} ; w_l = \frac{l}{j+k+l+m} ; w_m = \frac{m}{j+k+l+m}$$

where  $w_j + w_k + w_l + w_m = 1$ .

We calculate the forecasting enrollments by the following Rules.

**Rule 1:** If  $A_i \rightarrow A_j$  is a FLG then  $Z(t) = m_j$ , where  $m_j$  is the mid value corresponding to the interval of  $A_j$ ,

and  $F(t) = Z(t) - |\text{Actual value of } A_j - m_j|$  where  $F(t)$  is the forecasted value of  $A_j$ .

**Rule 2 :** Let  $A_i \rightarrow A_j^{(a)}, A_k^{(b)}, A_l^{(1)}, \dots$  be a FLG where  $A_j$  and  $A_k$  are repeated for a and b times respectively etc.

Let  $Z_1(t) = [a \text{ times of } m_j, b \text{ times of } m_k, \dots][a \text{ times of } w_j, b \text{ times of } w_k, \dots]^T$   
 and let  $Z_2(t) = [\text{Actual value} - \text{Average}(m_j, m_k, m_l \text{ etc})]$ . Here actual values corresponding to  $A_j$  or  $A_k$  or  $A_l \dots$

Then  $F(t) = Z_1(t) + Z_2(t)$

In the following, we apply the proposed method to forecast the enrollment of the University of Alabama shown in Table !. The length of interval, the intervals and the midpoints of the intervals are as follows:

$$D_{min} = 13055, D_{max} = 19337.$$

$$D_1 = 55, D_2 = 663 \text{ and Define } U = [13000, 20000]$$

Actual enrollment in an ascending sequence:  
 13055,13563,13867,14696,15145,15163,15311,15433,15460,15497,15603,  
 15861,15984,16388,16807,16859,16919,18150,18876,18970,19328, 19337.

Differences:

508,304,829,449,18,148,102,27,37,106,258,123,404,19,52,60,1231,726,94, 358,9.

When we perform the above steps we can get the length of interval as **57.76** and also we get the following intervals:

$$u_1 = [ 13055,13112.76],$$

$$u_2 = [ 13112.76, 13170.52], \dots$$

After calculating the midpoint of each interval  $u_i$  where  $1 \leq i \leq 108$ , we get the results as follows.

$$m_1 = 13084,$$

$$m_2 = 13141.64, \dots$$

**Table 1:**Fuzzified and Forecasted enrollments of the University of Alabama.

Year	Actual Enrollment ( $A_v$ )	Fuzzified Enrollment	Forecasted Enrollment ( $F_v$ )
1971	13055	$A_1$	-
1972	13563	$A_9$	13537
1973	13867	$A_{14}$	13815
1974	14696	$A_{29}$	14704
1975	15460	$A_{42}$	15464
1976	15311	$A_{39}$	15315
1977	15603	$A_{44}$	15569
1978	15861	$A_{48}$	15826
1979	16807	$A_{65}$	16808
1980	16919	$A_{67}$	16920.5
1981	16388	$A_{58}$	16395
1982	15433	$A_{41}$	15391
1983	15497	$A_{42}$	15443
1984	15145	$A_{36}$	15148
1985	15163	$A_{37}$	15171
1986	15984	$A_{51}$	15988
1987	16859	$A_{66}$	16863
1988	18150	$A_{88}$	18145
1989	18970	$A_{102}$	18951
1990	19328	$A_{108}$	19292
1991	19337	$A_{108}$	19343
1992	18876	$A_{101}$	18882.5

Then the definitions of the fuzzy sets  $A_1, A_2, \dots, A_{108}$  are as follows:

$$A_1 = \{ \frac{1}{13055}, \frac{0}{13055}, \dots, \frac{0}{13055} \}$$

$$A_2 = \{ \frac{0}{13055}, \frac{1}{13055}, \dots, \frac{0}{13055} \}$$

$$A_3 = \{ \frac{0}{13055}, \frac{0}{13055}, \frac{1}{13055}, \dots, \frac{0}{13055} \}$$

$$\vdots$$

$$\vdots$$

$$A_{108} =$$

**Fuzzy logical relationships:**

$$\begin{aligned}
 & A_1 \rightarrow A_9 : A_9 \rightarrow A_{14} : \quad A_{14} \rightarrow A_{29} : A_{29} \rightarrow A_{42} : \quad A_{42} \rightarrow A_{39} : \\
 & A_{39} \rightarrow A_{44} : \quad A_{44} \rightarrow A_{48} : \quad A_{48} \rightarrow A_{65} : \quad A_{65} \rightarrow A_{67} : \quad A_{67} \rightarrow A_{58} : \\
 & A_{58} \rightarrow A_{41} : A_{41} \rightarrow A_{42} : \quad A_{42} \rightarrow A_{36} : \quad A_{36} \rightarrow A_{37} : A_{37} \rightarrow A_{51} : \\
 & A_{51} \rightarrow A_{66} : A_{66} \rightarrow A_{88} : \quad A_{88} \rightarrow A_{102} : \quad A_{102} \rightarrow A_{108} : \quad A_{108} \rightarrow A_{108} :
 \end{aligned}$$

$$A_{108} \rightarrow A_{101} :$$

**Fuzzy logical relationship group:**

$$\begin{aligned}
 & A_1 \rightarrow A_9 : A_9 \rightarrow A_{14} : \quad A_{14} \rightarrow A_{29} : \\
 & A_{29} \rightarrow A_{42} : A_{42} \rightarrow A_{39}^{(1)}, A_{36}^{(1)} : A_{39} \rightarrow A_{44} : \\
 & A_{44} \rightarrow A_{48} : A_{48} \rightarrow A_{65} : \quad A_{65} \rightarrow A_{67} : \\
 & A_{67} \rightarrow A_{58} : A_{58} \rightarrow A_{41} : A_{41} \rightarrow A_{42} : \\
 & A_{36} \rightarrow A_{37} : A_{37} \rightarrow A_{51} : A_{51} \rightarrow A_{66} : \\
 & A_{66} \rightarrow A_{88} : A_{88} \rightarrow A_{102} : \quad A_{102} \rightarrow A_{108} : \\
 & A_{108} \rightarrow A_{108}^{(1)} A_{101}^{(1)}
 \end{aligned}$$

**Experimental Results**

To evaluate the forecasting performance, four fuzzy time series methods are adopted for comparing of their forecasting results with those obtained by proposed method. The mean absolute percentage error (MAPE) is used to evaluate the forecasting result accuracy.

The formula is:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{A_{t+1} - A_t}{A_t} \right| \times 100$$

Index	Chen(1996)	Hwang et.al(1998)	Lee and Chou (2004)	Proposed method
MAPE	3.08	2.94	2.69	.51

**Conclusion**

In this paper, we have developed a new fuzzy time series forecasting method for forecasting enrollment of the University of Alabama based on interval length, weightage factor and fuzzy logical relationships. In other words the proposed method gets a higher accuracy rate than Chen(1996), Hwang et.al(1998), Lee(2004).The rate of accuracy corresponds to minimizing the length of intervals and maximizing the number of intervals.

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